IMPLICIT DYNAMIC ROUTING OF FLOODS AND SURGES IN THE LOWER MISSISSIPPI[1]

by

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ABSTRACT. In streamflow forecasting, transient stages and discharges are computed for various forecast points along a river from a given (predicted or observed) stage or discharge hydrograph at either the upstream extremity of the river reach as in the case of a flood wave propagating in the downstream direction, or at the downstream extremity as in the case of a tidal or hurricane surge propagating in the upstream direction. The stages and discharges may be computed by an implicit dynamic routing technique in which the complete one-dimensional differential equations of unsteady flow are solved by an implicit four-point finite difference technique which necessitates the solution of successive systems of nonlinear equations. An extrapolation technique along with a special quad-diagonal Gaussian elimination procedure are used in conjunction with the Newton-Raphson method to provide a very efficient solution technique for the nonlinear systems. The implicit dynamic routing technique is applied to some recent floods and hurricane surges which have occurred in the lower portion of the Lower Mississippi River. The computed and observed stages are compared, and the results of numerical experiments are presented which illustrate the effects of the computational time step size, the Manning roughness coefficient, and the various acceleration terms in the equation of dynamic equilibrium.

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INTRODUTION

<u>Background</u>. The complete one-dimensional partial differential equations of unsteady flow, consisting of an equation for conservation of mass and an equation for conservation of momentum, form a basis for constructing very general and powerful mathematical models which can be used in streamflow forecasting to compute transient stages and discharges at various points along a river. Among the types of transient flow which may be simulated using these models are flood waves which propagate downstream in rivers with very mild bottom slopes and hurricane or tidal surges which propagate upstream against the normal river flow. In each case, the dynamic inertia and pressure forces can interact significantly with the friction forces to produce the spatial and temporal changes which the wave undergoes as it propagates through the river.

Due to the complexity of the complete unsteady flow equations, many of the mathematical models which have been used in the past and those which are still being used in operational streamflow forecasting are based primarily on the equation for conservation of mass while the equation for conservation of momentum is either completely ignored or greatly simplified to include only the effects of the friction forces. Such simplifications can cause inaccuracies in the computed stages and discharges, particularly when the wave movement is influenced by the neglected inertia and pressure forces as in the examples of wave movement cited above. Many investigations have been conducted in recent years to solve the complete unsteady flow equations via numerical integration of finite difference expressions of the differential equations. Beginning with the pioneering work of Stoker and his colleagues [Stoker, 1957] who used an explicit finite difference technique to solve the unsteady flow equations for routing of Ohio River floods, many others have investigated unsteady flows using the explicit method, e.g., Liggett and Woolhiser [1967], Dronkers [1969], and Garrison, <u>et</u> <u>al</u>. [1969]. However, numerical stability considerations restrict this method to very small computational time steps (on the order of a few minutes or even seconds). For this reason, the explicit method is very inefficient for the computation of floods occurring in large rivers and having durations of days or even weeks. To overcome this disadvantage, implicit finite difference techniques which have no restrictions in the size of the time step due to numerical stability have been investigated recently, e.g., Abbott and Ionescu [1967], Dronkers [1969], Balloffet [1969], Baltzer and Lai [1969], Amein and Fang [1970], Contractor and Wiggert [1972], Quinn and Wylie [1973], and Chaudhry and Contractor [1973].

<u>Purpose and scope</u>. This paper presents an implicit four-point finite difference technique for solving the unsteady flow equations in order to compute stages and discharges for flood waves in rivers having very mild bottom slopes and for hurricane surges propagating upstream against a downstream flow. The mathematical model is applied to the above types of transient flow which have occurred during recent years in the Lower Mississippi River. The computed and observed stages are compared, and numerical experiments are conducted to examine the effects of the computational time step size and the Manning roughness coefficient, and to determine the significance of the inertia and pressure terms relative to the friction term in the equation for conservation of momentum.

General description of study reach. The portion of the Lower Mississippi River of concern in this investigation is a 291.7 mile reach extending from the Red River Landing gage (RM*302.4) downstream to the Venice gage (RM 10.7). Among the 14 continuous and intermittent gaging stations within the study reach, those at Baton Rouge (RM 228.4), Donaldsonville (RM 175.4), Reserve (RM 138.7), New Orleans, Carrollton (RM 102.8), Chalmette (RM 91.0) and Pointe A La Hache (RM 48.7) are used to evaluate the computations. This reach of the Lower Mississippi is confined within levees for most of its length, although some overbank flow occurs along portions of the upper 70 miles. Throughout the study reach, the alluvial river meanders between deep bends and relatively shallow crossings; the sinuosity coefficient is approximately 1.6. The low flow depth varies from a minimum of 30 ft at some crossings to a maximum bend depth of almost 200 ft. The average width is approximately one-half mile. The average channel bottom slope for the study reach is a very mild 0.0000064 ft/ft (0.034 ft/mi). The discharge varies from low flows of about 120,000 cfs to flood discharges of over 1,200,000 cfs.

MATHEMATICAL MODEL

Unsteady flow equations. The one-dimensional differential equations of unsteady flow are:

$$\frac{\partial A}{\partial t} + \frac{\partial (AV)}{\partial x} = 0 \tag{1}$$

$$\frac{1}{g}\frac{\partial V}{\partial t} + \frac{1}{2g}\frac{\partial V^2}{\partial x} + \frac{\partial h}{\partial x} + S_f = 0 \tag{2}$$

Equation 1 is known as the continuity equation and conserves the mass of the flow, while (2) conserves the momentum of the flow and is known as the equation of motion or dynamic equilibrium. In the above equations, x is the distance along the channel, t is time, A is the wetted cross-sectional area, V is the average velocity, h is the water surface elevation, S_f is the friction slope, and g is the acceleration due to gravity. The first two terms in (2) are the inertia terms; the first is the local acceleration slope and the second is the convective acceleration slope. The third term is the water surface slope which is equivalent to the depth gradient less the channel bottom slope, S_o. A derivation of the unsteady flow equations may be found in Stoker [1957], Chow [1959], and Henderson [1966].

Equations 1 and 2 constitute a system of first order, quasi-linear, partial differential equations of the hyperbolic type. They contain two independent variables, x and t, and two dependent variables, h and V; the remaining terms are either functions of x, t, h and/or V, or they are constants. These equations are not amendable to analytical solutions except in cases where the channel geometry is uncomplicated and the nonlinear properties

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*River Mile referenced from zero point at Head of Passes, La.

of the equations are either neglected or made linear. The equations may be solved numerically by performing two basic steps. First, the partial differential equations are represented by a corresponding set of finite difference algebraic equations; and second, the system of algebraic equations are solved in conformance with prescribed initial and boundary conditions.

Implicit finite difference equations. Equations 1 and 2 are approximated by implicit four-point finite difference expressions, and the continuous x-t region in which solutions of h and V are sought is represented by a rectangular net of discrete points as shown in Figure 1. The net points are determined by the intersection of lines drawn parallel to the x and t axes. Those parallel to the x axis represent time lines; they have a spacing of Δt which need not be constant. Those parallel to the t axis represent discrete locations along the river (x axis); they have a spacing of Δx which also need not be constant. Each point in the rectangular network can be identified by a subscript (m) which designates the x position and a superscript (n) which designates the time line.

Using a generalized implicit four-point finite difference scheme, the time derivatives are approximated by a forward difference quotient centered between the m^{th} and m+1 points along the x axis, i.e.,

$$\frac{\partial K}{\partial t} \simeq \frac{K_m^{n+1} + K_{m+1}^{n+1} - K_m^n - K_{m+1}^n}{2\ \Delta t} \tag{3}$$

where K represents any variable. The spatial derivatives are approximated by a forward difference quotient positioned between two adjacent time lines according to weighting factors of θ and $1-\theta$, i.e.,

$$\frac{\partial K}{\partial x} \simeq \theta \left(\frac{K_{m+1}^{n+1} - K_m^{n+1}}{\Delta x}\right) + (1-\theta) \left(\frac{K_{m+1}^n - K_m^n}{\Delta x}\right) \tag{4}$$

Variables or functions other than derivatives are approximated at the time level where the spatial derivative is evaluated by using the same weighting factors, i.e.,

$$K \simeq \theta \left(\frac{K_m^{n+1} + K_{m+1}^{n+1}}{2}\right) + (1-\theta) \left(\frac{K_m^n + K_{m+1}^n}{2}\right) \tag{5}$$

A weighting factor of θ=1.0 yields the forward fully implicit scheme used by Baltzer and Lai [1969], Dronkers [1969], and examined by Gunaratnam and Perkins [1970]. A weighting factor of θ=0.5 yields the box scheme suggested by Thomas [1934], and used by Amein and Fang [1970], Contractor and Wiggert [1971], and Fread [1973b]. The influence of the θ weighting factor on the accuracy of the computations was discussed by Fread [1973a, 1974] who concluded that the accuracy decreases as θ departs from 0.5 and approaches 1.0. This effect becomes more pronounced as the magnitude of the computational time step increases. In this paper, a weighting factor of 0.55 is used so as to minimize the loss of accuracy associated with greater θ values while avoiding the possibility of a weak or pseudo-instability noticed by Baltzer and Lai [1969], Quinn and Wylie [1973], and Chaudhry and Contractor [1973] when a θ of 0.5 is used.

When the finite difference operators defined by equations (3), (4), and (5) are used to replace the derivatives and other variables in (1) and (2), the following implicit four-point difference equations are obtained:

$$\frac{A_m^{n+1} + A_{m+1}^{n+1} - A_m^n - A_{m+1}^n}{2\,\Delta t} + \theta\left[\frac{(AV)_{m+1}^{n+1} - (AV)_m^{n+1}}{\Delta x}\right] +$$

$$(1-\theta)\left[\frac{(AV)_{m+1}^{n} - (AV)_m^{n}}{\Delta x}\right] = 0 \tag{6}$$

$$\frac{V_m^{n+1} + V_{m+1}^{n+1} - V_m^n - V_{m+1}^n}{2\,\Delta t} + \theta\left[\frac{(V^2)_{m+1}^{n+1} - (V^2)_m^{n+1}}{\Delta x}\right. +$$

$$\left. g\,\frac{(h_{m+1}^{n+1} - h_m^{n+1})}{\Delta x} + g\,\bar{S}_f^{n+1}\right] + (1-\theta)\left[\frac{(V^2)_{m+1}^{n} - (V^2)_m^{n}}{\Delta x} + g\,\frac{(h_{m+1}^{n} - h_m^{n})}{\Delta x}\right. +$$

$$\left. g\,\bar{S}_f^{n}\right] = 0 \tag{7}$$

where:

$$\bar{S}_f = \frac{\bar{n}^2\,|\bar{V}|\,\bar{V}}{2.208\,\bar{R}} \tag{8}$$

in which,

$$\bar{V} = \frac{V_m + V_{m+1}}{2} \tag{9}$$

by a compact quad-diagonal Gaussian elimination algorithm [Fread, 1971] which is very efficient with respect to computing time and storage.

The efficiency of the method is quite dependent on the success with which the first trial values are made. A method of parabolic extrapolation has been found to provide trial values sufficiently close to the unknowns to assure convergence within one to three iterations.

DATA REQUIREMENTS

Cross-sectional geometry. Cross-sectional properties were obtained from a 1963 hydrographic survey of the Mississippi River [U. S. Army Engr. Dist., New Orleans, 1965]. Individual cross sections were plotted at all sections such as crossings and entrances, midpoints, and exits of bends where the width and/or depth were judged to change sufficiently to violate the assumption of linear variation between adjacent cross sections. The cross sectional properties at each section are described by a step function of the water surface width, B, as a function of the water surface elevation, h_k. In this paper, h_1 is mean sea level. From this basic data, cross-sectional widths and areas pertaining to any water surface elevation, h_n, above h_1 can be computed by the following:

$$B_n = B_k + \left(\frac{h_n - h_k}{h_{k+1} - h_k}\right) (B_{k+1} - B_k) \tag{11}$$

$$A_n = A_k + \left(\frac{B_n - B_k}{2}\right) (h_n - h_k) \tag{12}$$

where,

$$h_k \leq h_n \leq h_{k+1} \tag{13}$$

and A_k is the total cross-sectional area associated with each h_k water surface elevation. Approximately six to eight values of h_k and B_k are sufficient to adequately describe the variation of B throughout the range of possible water surface elevations at each section.

Cross sections are used at each of 13 continuous or intermittent gaging stations along the 291.7 mile study reach. Also, an average of the individual cross sections is computed for each point midway between adjacent gaging stations. The average cross sections are weighted using the distance between individual cross sections as weighting factors. Thus, a total of 25 cross sections are used for computational points along the study reach. The computational distance intervals are unequal, ranging from approximately 6 to 21 miles.

Upstream boundary. For the case of a flood wave propagating downstream, the upstream boundary consists of mean daily water surface elevations at Red River Landing (RM 302.4). These are input as tabular values at 24 hr intervals. Intermediate values are obtained via linear interpolation.

$$\bar{R} = \left(\frac{A_m + A_{m+1}}{B_m + B_{m+1}}\right)^{4/3} \qquad (10)$$

and $\bar{n}$ is Manning's roughness coefficient while B is the width of the cross section at the elevation of the water surface.

Equations 6 and 7 constitute a system of algebraic equations which are nonlinear with respect to the unknowns, i.e., the values of the dependent variables h and V at the net points m and m+1 at the time line designated as n+1. The terms associated with the n^{th} time line are known from either the initial conditions or previous computations.

Initial conditions. The initial conditions refer to the values of h and V associated with each point along the x axis for the first time line. They are obtained from: 1) a previous unsteady flow solution; 2) a step-wise steady gradually varied flow backwater computation; or 3) judicious estimates which will converge to the true initial conditions when the unsteady flow equations are solved with the boundary conditions held constant during the first several time steps. The last method is used for the computations presented herein.

Boundary conditions. The boundary conditions consist of a description of either water surface elevation (h) or discharge (AV) as a function of time at the upstream and downstream extremities of the study reach. The downstream boundary may also be a specified relationship between h and AV such as an empirical rating curve, weir-type flow, or normal depth-discharge relationship corrected for unsteady effects. In this paper all flows are subcritical, a condition which requires that one boundary condition be prescribed for the upstream extremity of the river and one for the downstream extremity.

Method of solution. Equations 6 and 7 cannot be solved in an explicit or direct manner for the unknowns since there are four unknowns and only two equations. However, if (6) and (7) are applied to each of the (M-1) rectangular grids between the upstream and downstream boundaries, a total of (2M-2) equations with 2M unknowns can be formulated. Then, prescribed boundary conditions, one at the upstream boundary and one at the downstream boundary, provide the necessary additional two equations required for the system to be determinate. The resulting system of 2M nonlinear equations with 2M unknowns is solved by a functional iterative procedure, the Newton-Raphson method [Isaacson and Keller, 1967]. Computations for the iterative solution of the nonlinear system are begun by assigning trial values to the 2M unknowns. Substitution of the trial values into the system of nonlinear equations yields a set of 2M residuals. The Newton-Raphson method provides a means for correcting the trial values until the residuals vanish or are reduced to tolerable magnitudes. A system of 2M x 2M linear equations relate the corrections to the residuals and to a coefficient matrix composed of partial derivatives of each equation with respect to each unknown variable in that equation. The coefficient matrix of the linear system has a banded structure which allows the system to be solved by a compact quad-diagonal Gaussian elimination algorithm [Fread, 1971] which is very efficient with respect to computing time and storage.

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Upstream boundary. For the case of a flood wave propagating downstream, the upstream boundary consists of mean daily water surface elevations at Red River Landing (RM 302.4). These are input as tabular values at 24 hr intervals. Intermediate values are obtained via linear interpolation.

For the case of a hurricane surge propagating upstream, the upstream boundary consists of a steady discharge (AV) at Red River Landing (RM 302.4). This boundary condition is an assumption, since the discharge at the upstream extremity is affected eventually by the propagating surge; however, it is adequate for determining the discharges and water surface elevations affected most by the hurricane surge at locations a considerable distance downstream, e.g., Chalmette (RM 91.0) and Carrollton (RM 102.8).

Downstream boundary. For the flood wave, the downstream boundary consists of mean daily water surface elevations at Venice (RM 10.7). These are input as tabular values at 24 hr intervals. Intermediate values are obtained via linear interpolation.

For the hurricane surge, the downstream boundary consists of hourly water surface elevations at Pointe A La Hache (RM 48.7). These are input as tabular values at hourly intervals.

Manning's roughness coefficients. In this paper, the 1963 spring flood of March 5 to April 26 is used to obtain representative roughness coefficients for the study reach. The observed stages and corresponding observed discharges are used in conjunction with a steady, gradually varied flow step-wise computation, similar to that reported by Fread and Harbaugh [1971], to determine the Manning $\bar{n}$ in an iterative fashion. Commencing with an observed stage at a downstream gaging station and an assumed value for $\bar{n}$, the backwater profile is computed in a step-wise manner until a selected upstream gaging station is reached. If the computed stage at the upstream section agrees with the observed stage, the assumed $\bar{n}$ value is considered correct; however, if they do not agree within an acceptable tolerance, $\bar{n}$ is changed and the profile is recomputed. This process is repeated until the computed upstream stages agree within the prescribed tolerance; the $\bar{n}$ used in this last computation is assumed to be the correct value for that particular reach and discharge. In this way, $\bar{n}$ is computed for a range of discharges for each reach. Linear step-functions describing the relationship between $\bar{n}$ and discharge are used in the computation of the friction slope, S_f, in (8).

COMPUTATION OF FLOODS PROPAGATING DOWNSTREAM

Calibration of model. The implicit dynamic routing model was calibrated using observed stages of the 1963 spring flood at selected stations throughout the Red River Landing (RM 302.4) - Venice (RM 10.7) reach. The calibration consisted of determining the functional relationship between $\bar{n}$ and discharge (AV) for each of the sub-reaches: Red River Landing - Baton Rouge, Baton Rouge - Donaldsonville, Donaldsonville - Carrollton, Carrollton - Pte. A La Hache, and Pte. A La Hache - Venice. The calibration was accomplished by a trial-and-error procedure in which trial n-AV relationships for each sub-reach were used in the implicit dynamic routing model to compute stages at the selected stations. Computational time steps of 24 hrs were used; this time step coincided with the resolution of the stage hydrographs at the upstream and downstream boundaries. The first trial n-AV relationships were those obtained for each sub-reach via the iterative application of the steady flow backwater computations described previously. Each n-AV relationship was adjusted until the rms* error between observed and computed stages at the selected stations was minimized. The needed adjustments proved to be minor, thus requiring only a few trials. Comparisons of the observed stages with the computed stages using the calibrated model are presented in Figures 2, 3, and 4 for six gaging stations within the study reach. The average rms error for the six stations is 0.32 ft.

Verification of the model. The 1969 spring flood of January 23 to March 27 was used to verify the calibrated model. The verification consisted of comparing observed stages with those computed using the same n-AV relationships determined during the calibration of the model. As in the calibration of the model, computational time steps of 24 hrs were used to verify the model. Comparisons of the observed and computed stages for the 1969 flood are presented in Figures 5, 6, and 7 for six selected stations within the study reach. The average rms error for the six stations is 0.46 ft. The model was also verified for floods occurring in 1966, 1968, and 1971 resulting in average rms errors of 0.50, 0.43, and 0.40 ft, respectively.

COMPUTATION OF SURGES PROPAGATING UPSTREAM

In August 1969, Hurricane Camille produced a strong storm tide in the Mississippi Delta area with water levels up to 12 ft. This high tide (hurricane surge) propagated into the Lower Mississippi River and traveled upriver several hundred miles. The implicit dynamic routing technique was used to compute the stages and discharges produced by the passage of the surge at Chalmette (RM 91.0) and Carrollton (RM 102.8). The downstream boundary condition was the observed hourly stage hydrograph at Pte. A La Hache (RM 48.7) and the upstream boundary condition was an assumed steady discharge of 253,000 cfs at Red River Landing (RM 302.4). The Manning roughness coefficients for the study reach were maintained the same as determined during the calibration of the 1963 spring flood.

The computed and observed stage hydrographs at Chalmette and Carrollton are shown in Figure 8. The rms errors between the computed and observed stage hydrographs are 0.33 ft. and 0.34 ft. for Chalmette and Carrollton, respectively. A computational time step of 1 hour was used; this compares to a maximum allowable time step of 0.15 hr which could be used in an explicit solution of the unsteady flow equations.

The computed discharge hydrograph for Carrollton is shown in Figure 9. The negative discharge is associated with flow in the upstream direction resulting from the passage of the hurricane surge at Carrollton. It is interesting to note that for a few hours after the main portion of the surge has passed Carrollton, the discharge in the downstream direction is slightly greater than before the surge arrived due to a sufficiently large transitory depth gradient produced by the surge while it is in the upstream vicinity of Carrollton.

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*root mean square

NUMERICAL EXPERIMENTS

Effect of time step size. The 1963 flood was routed through the study reach using different size time steps. The resulting stage hydrographs at Donaldsonville are shown in Figure 10. There is little noticeable difference between the hydrographs computed with 1, 24, and 48 hr time steps; however, as the time step continues to increase in size the computed hydrographs depart further from those computed with the smaller time steps. Since the accuracy of the computations [Fread, 1973a and 1974] as well as the required computation time decrease as the time step increases in size, there exists a trade-off between efficiency and accuracy which is controlled by the selection of the computational time step size. Such flexibility is a desirable advantage of the implicit dynamic routing method while the explicit method is restricted to very small time steps in order to satisfy numerical stability constraints. For comparison, the maximum permissible time step which could be used by an explicit method to compute the 1963 flood is 0.15 hr. The ratio of the implicit time step to the explicit time step is known as the Courant number, C_n. A summary of the effects of the size of the computational time step as to computational efficiency and accuracy of the implicit method for routing the 1963 flood through the study reach is presented in Table 1.

Effect of the Manning roughness coefficient. A knowledge of the effects of Manning's $\bar{n}$ on the computed stages is essential for achieving an efficient and accurate calibration of the implicit dynamic routing model. The computed stages are affected quite significantly by the particular values of Manning's $\bar{n}$ that are used in the computations. This is illustrated by the stage hydrographs for Donaldsonville shown in Figure 11. Using the 1963 flood and the calibrated $\bar{n}$ values for all reaches from Red River Landing (RM 302.4) to Venice (RM 10.7), the computed stage hydrograph for Donaldsonville is shown in Figure 11 as the middle hydrograph which is denoted by $\bar{n}_o$. Upon increasing $\bar{n}_o$ by 20 percent for the reach from Donaldsonville to Carrollton and repeating the computations, the resulting stages for Donaldsonville, shown in Figure 11, are seen to be higher than those computed using $\bar{n}_o$. The $\bar{n}$ values for the same reach are also decreased by 20 percent and the computations repeated. The Donaldsonville stage hydrograph computed using the decreased $\bar{n}$ values is shown in Figure 11 to be lower than the stage hydrograph computed using $\bar{n}_o$. The 20 percent variation is $\bar{n}_o$ results in approximately a 14 percent variation in the computed stage hydrographs at Donaldsonville. Although a one to one relationship between a change in Manning's $\bar{n}$ and the resulting change in the computed stage does not exist for the Donaldsonville gage; nevertheless, the effect of altering $\bar{n}$ is quite significant.

When the $\bar{n}$ values vary from reach to reach along a river, as found to be the case for the lower portion of the Lower Mississippi River, changes in the $\bar{n}$ values for any one reach produce varying changes in the computed stages at all points along the river. This effect is shown in Figure 12. In the case of a 20 percent increase in the friction for the Donaldsonville - Carrollton reach, the rms variation in the stage hydrographs varies from positive to negative depending upon the location of the gage in question with respect to the reach of river for which the $\bar{n}$ values are increased. The most significant change is an increase in the stages at locations a short distance upstream and downstream of Donaldsonville. However, the effect vanishes at locations far upstream and downstream of Donaldsonville. Also, it should be noted that the variation of the rms of the stage hydrograph changes from a positive effect to a negative effect at a particular location within the reach in which $\bar{n}$ is increased. Quite similar but opposite effects are produced in the rms of the stage hydrographs when the friction of the Donaldsonville - Carrollton reach is decreased by 20 percent.

Significance of terms in the equation of motion. The significance of the various terms in the equation of motion or dynamic equilibrium can be assessed by comparing their magnitudes. Such a comparison is given in Table 2 for a gradually varying transient (the 1963 flood in the Lower Mississippi) and for a more rapidly varying transient (the 1969 Hurricane Camille in the Lower Mississippi). In the former, the terms are evaluated at Reserve for both the rising limb and peak of the flood, while in the latter the terms are evaluated at Carrollton for both the rising limb and peak of the hurricane surge.

For the case of the slowly varying transient, it is evident that the local acceleration term is negligible compared to the friction slope term. However, the convective acceleration term is about 23 percent of the friction slope while the water surface slope has a magnitude of about 80 percent of the friction slope. Thus, the inclusion of the water surface slope term is considered essential while the inclusion of the convective acceleration term is of questionable value. Also, in Table 2 it is seen that the friction slope is 930 to 2,000 percent of the effective channel bottom slope at Reserve. This implies that a kinematic routing model, e.g., Brakensiek [1965] and Harley et al. [1970], which assumes the friction slope to be identical with the bottom slope would be in serious error. Likewise, hydrologic routing methods such as the Muskingum method which are derived from the same basic assumption [Cunge, 1969] would be of questionable value. Since the water surface slope term must be included, it would seem that the least complicated routing model for slowly varying transients in the lower portion of the Lower Mississippi River would be a diffusion wave approximation [Brakensiek, 1965] in which the following simplified equation of motion is used:

$$\frac{\partial h}{\partial x} + S_f = 0 \qquad (14)$$

However, if an implicit finite difference formulation is used so as to eliminate problems of numerical stability and permit large computational time steps which are essential for efficient computation of slowly varying transients, only about a 20 percent savings in computer time would be realized [Sevuk, 1973]. In view of this and the fact that the dynamic routing model used herein uses the complete form of the equation of motion such that the questionable convective acceleration term is not neglected, the implicit dynamic routing model is considered to be the appropriate model for computation of slowly varying transients in rivers of extremely mild channel bottom slopes.

For the case of the hurricane surge, the local acceleration term is quite significant, especially for the rising limb where its magnitude is 400 percent of the friction slope. As in the case of the 1963 flood computation, the convective acceleration term is of questionable significance since its magnitude is only about 12 percent of the friction slope. The water surface slope is quite significant, varying from 144 to 528 percent of the friction slope. Also, the friction slope is about 214 percent of the channel bottom slope. Thus, the relative magnitudes of the terms in the equation of motion indicate that the only term of questionable significance is the convective acceleration term; however, inclusion of this term does not materially increase the computation time. Hence, the implicit dynamic routing model which uses the complete equation of motion is considered the appropriate model for computation of hurricane surges in the Lower Mississippi River.

SUMMARY AND CONCLUSIONS

An implicit four-point finite difference technique was used to solve the complete one-dimensional equations of unsteady flow in conformance with initial and boundary conditions. The implicit dynamic routing model was calibrated and tested using data from floods propagating downstream and a hurricane surge propagating upstream through a 292 mile reach of the Lower Mississippi River.

The model proved to be very efficient computationally, e.g., routing a 56 day flood through the 292 mile study reach required less than 10 sec of CDC 6600 computer time. The computed stages compared favorably with observed stages at several gaging stations located along the study reach. The implicit dynamic routing model is flexible in that it permits the following:

1) computational time steps are not limited in size by numerical stability problems; therefore, they can be chosen so as to effect a suitable compromise between required computation time and acceptable accuracy;

2) computational distance steps can be unequal which is generally necessary when modeling natural channels of irregular geometry; and

3) transient waves which propagate downstream and/or upstream can be modeled since the inertia, pressure, friction, and gravity forces are included in the equation for conservation of momentum.

The implicit dynamic routing model is sensitive to the values selected for the Manning roughness coefficients. These should be determined from stage-discharge data via a combination of steady flow backwater computations and trial-and-error calibration runs using the dynamic routing model.

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Table 1. Effect of Computational Time Step Size on Computed Stage Hydrograph for 1963 Spring Flood at Donaldsonville

Δt (hr)	Courant Number C_n	Total Computation Time for 56 Day Transient Routed Through 24 Δx Finite Reaches (sec)	Computation Time per Day of Transient Routed per Finite Reach (sec/day/reach)	Donaldsonville Stage, rms Error Based on Stage Computed with Δt of 1 hr. (ft)
1	6.7	50	0.0373	0.00
24	160	9	0.0067	0.05
48	320	3.7	0.0028	0.09
96	640	2.7	0.0020	0.23
240	1600	2.0	0.0015	0.48

Table 2. Comparison of Terms in Equation of Motion

Location	Transient Flow Description	S_f (ft/ft)	S_o (ft/ft)	S_f/S_o (%)	$\frac{1}{g}\,\partial v/\partial t$ (ft/ft)	$\frac{1}{gS_f}\,\partial v/\partial t$ (%)	$\frac{1}{2g}\,\partial v^2/\partial x$ (ft/ft)	$\frac{1}{2gS_f}\,\partial v^2/\partial x$ (%)	$\partial h/\partial x$ (ft/ft)	$\frac{\partial h/\partial x}{S_f}$ (%)
Reserve RM 138.7	1963 Flood Rising Limb	.0000130	.0000014	930.	-.0000002	1.5	-.0000027	22.	-.0000106	82.
Reserve RM 138.7	1963 Flood Peak	.0000280	.0000014	2000.	-.0000000	0.0	-.0000067	24.	-.0000217	78.
Carrollton RM 102.8	1969 Surge Rising Limb	-.0000028	.0000014	200.	-.0000112	400.	.0000003	11.	.0000148	528.
Carrollton RM 102.8	1969 Surge Peak	-.0000032	.0000014	228.	-.0000010	31.	.0000004	12.	.0000046	144.

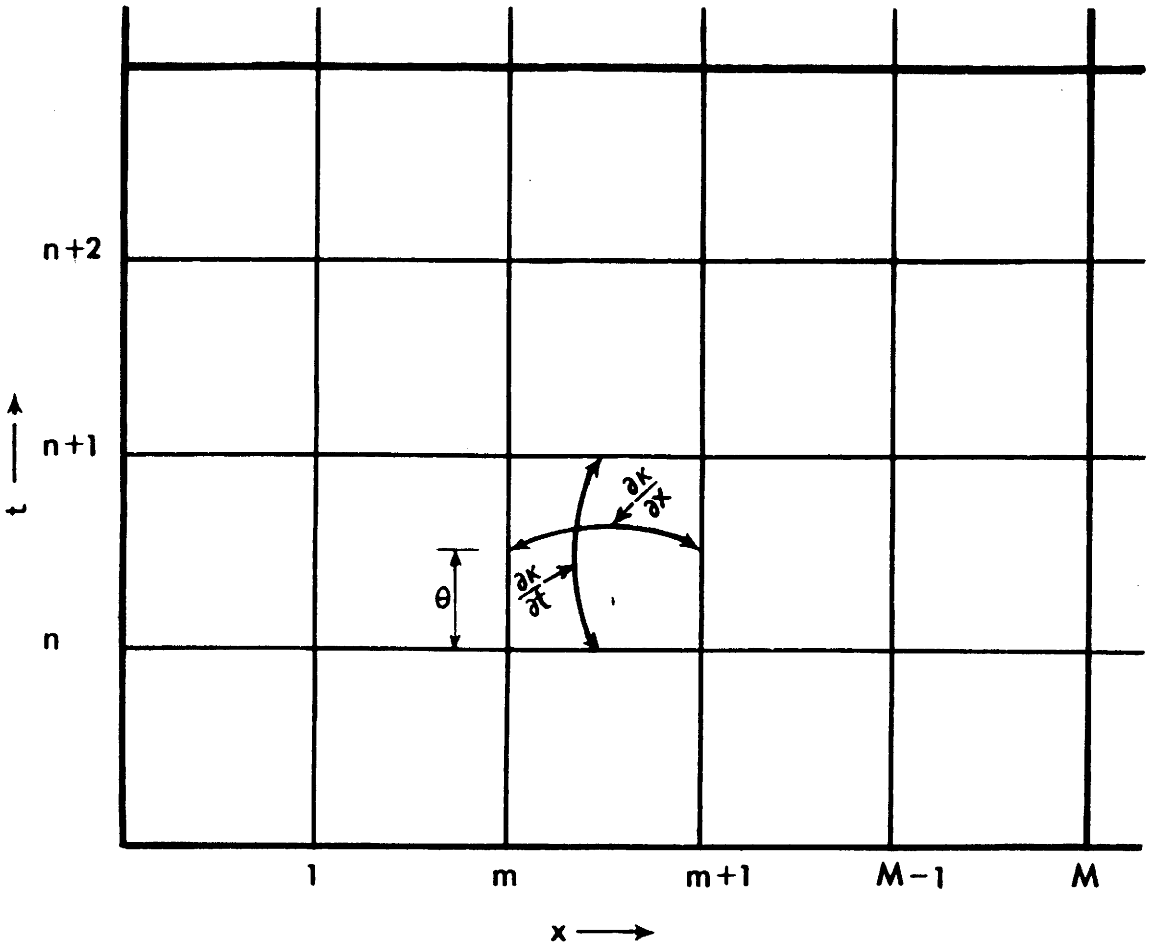


Figure 1. x-t Solution Region.

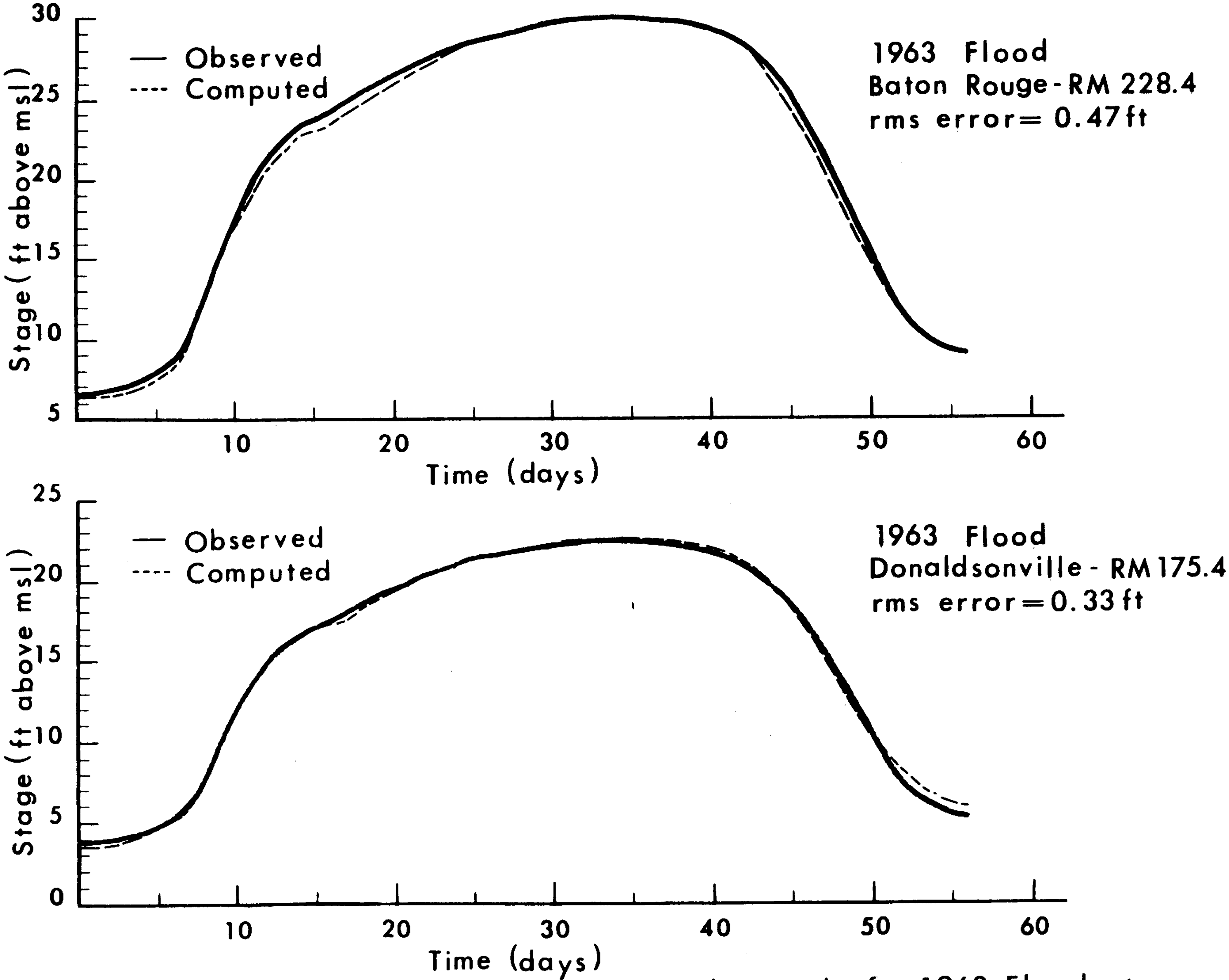


Figure 2. Computed and Observed Stage Hydrographs for 1963 Flood at Baton Rouge and Donaldsonville.

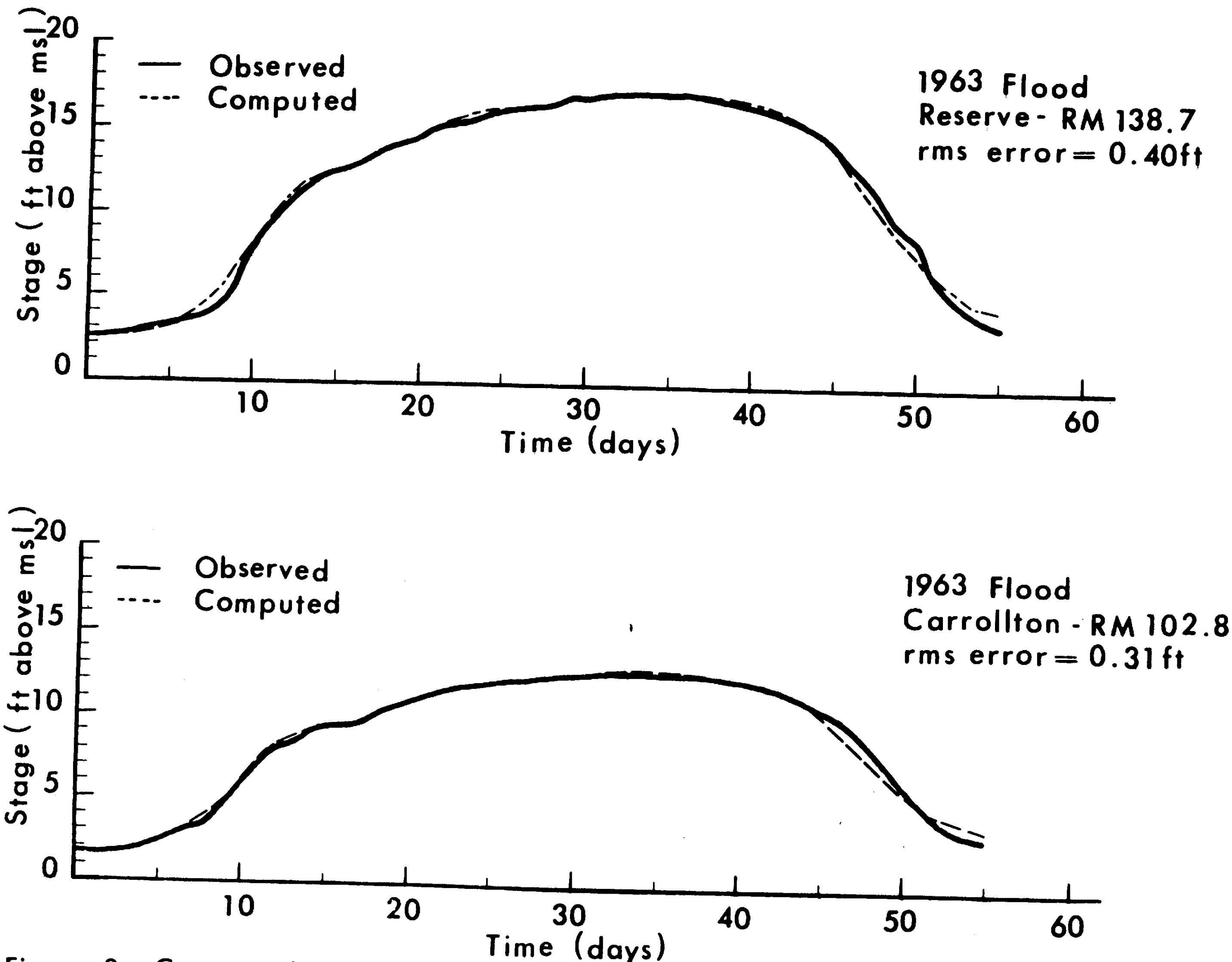
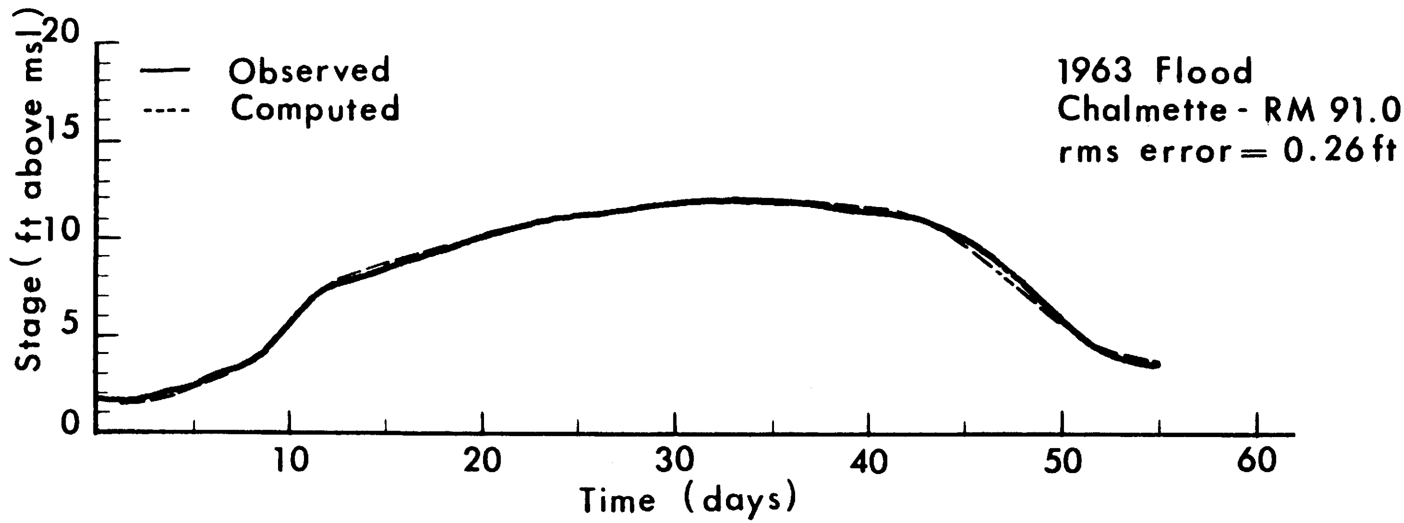


Figure 3. Computed and Observed Stage Hydrographs for 1963 Flood at Reserve and Carrollton.



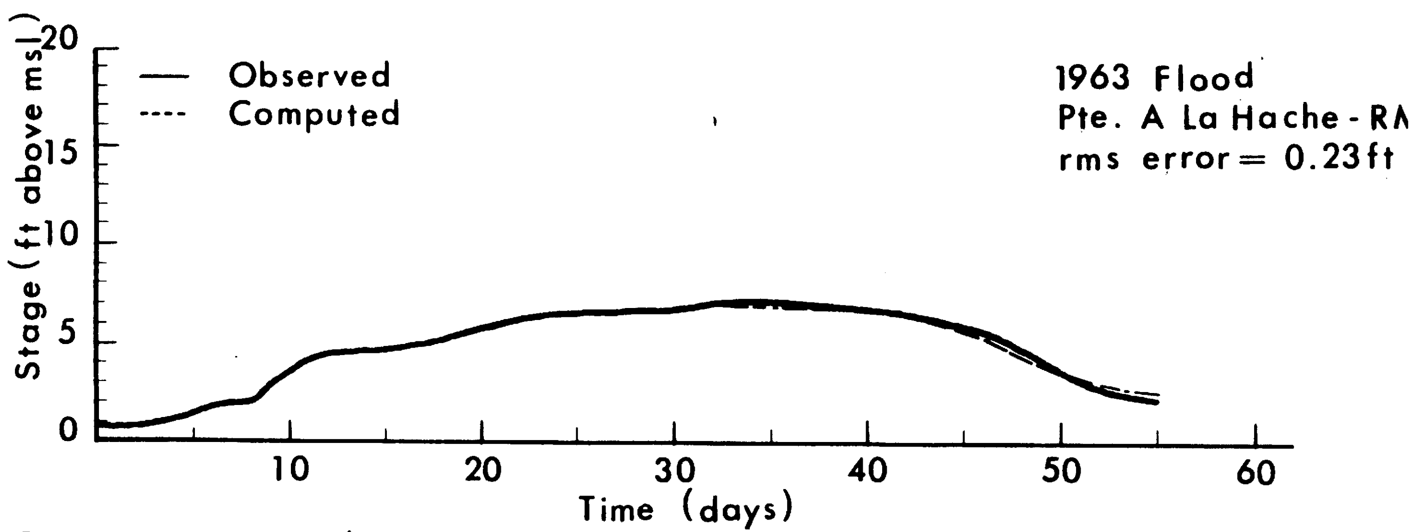
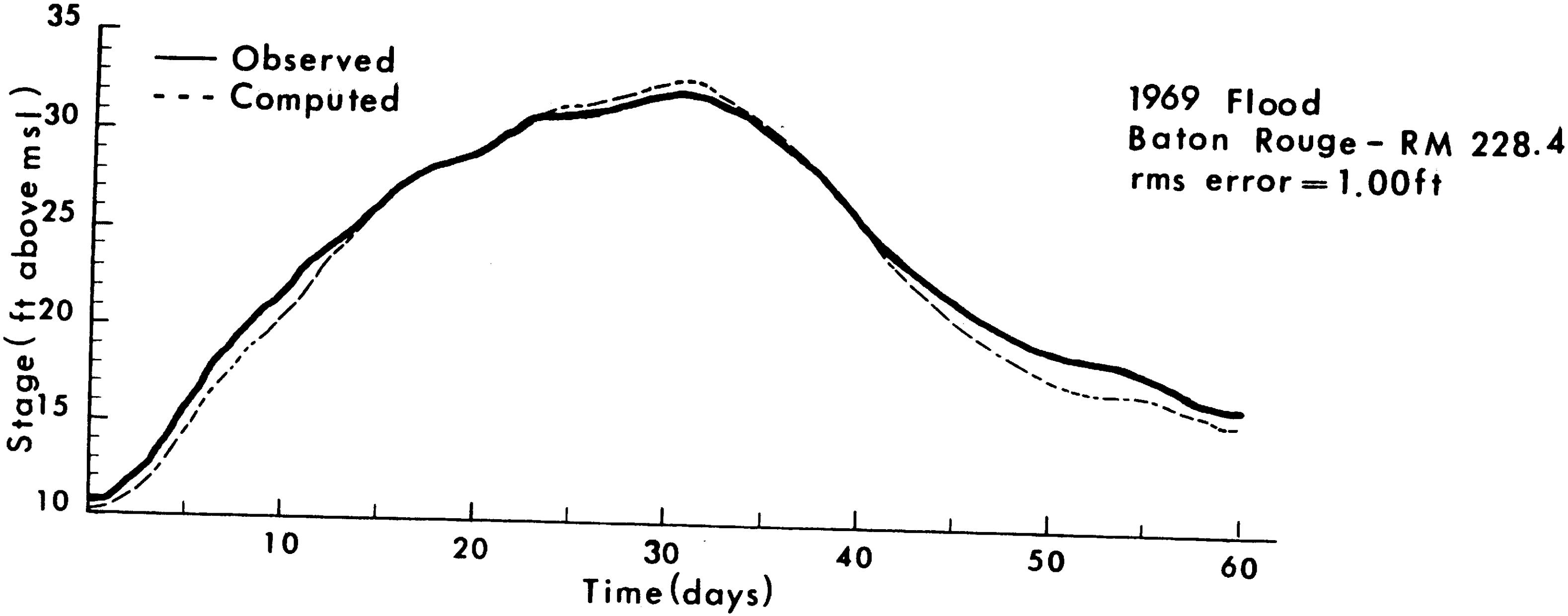


Figure 4. Computed and Observed Stage Hydrographs for 1963 Flood at Chalmette and Pte. A La Hache.



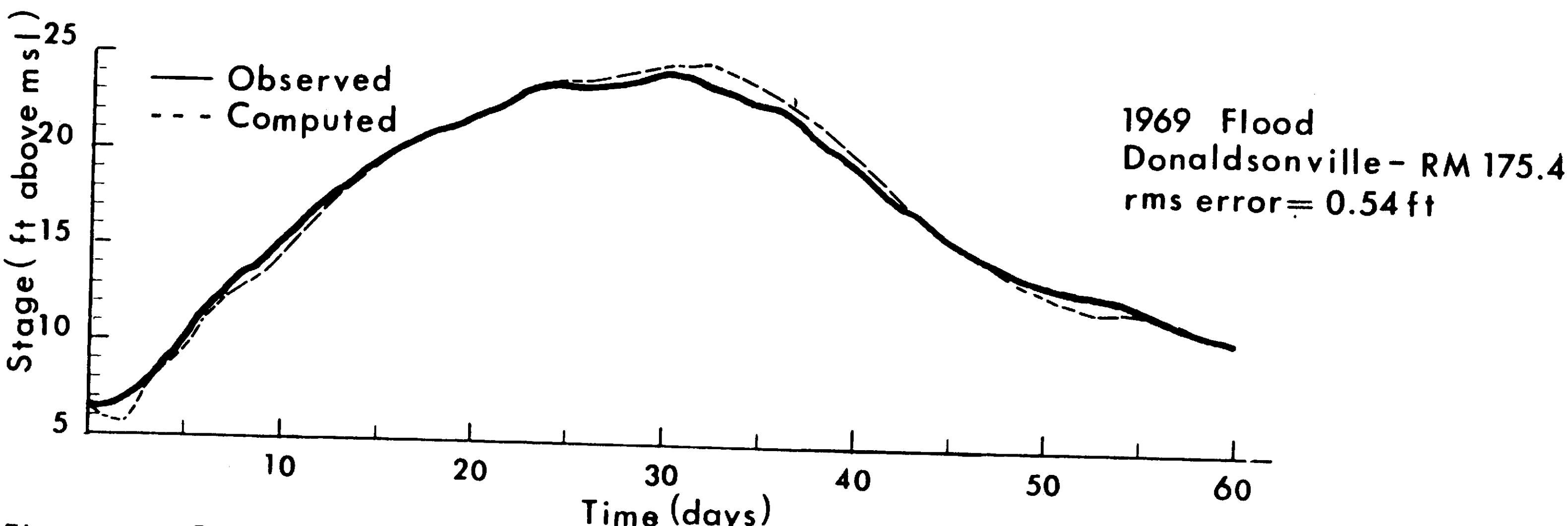


Figure 5. Computed and Observed Stage Hydrographs for 1969 Flood at Baton Rouge and Donaldsonville.

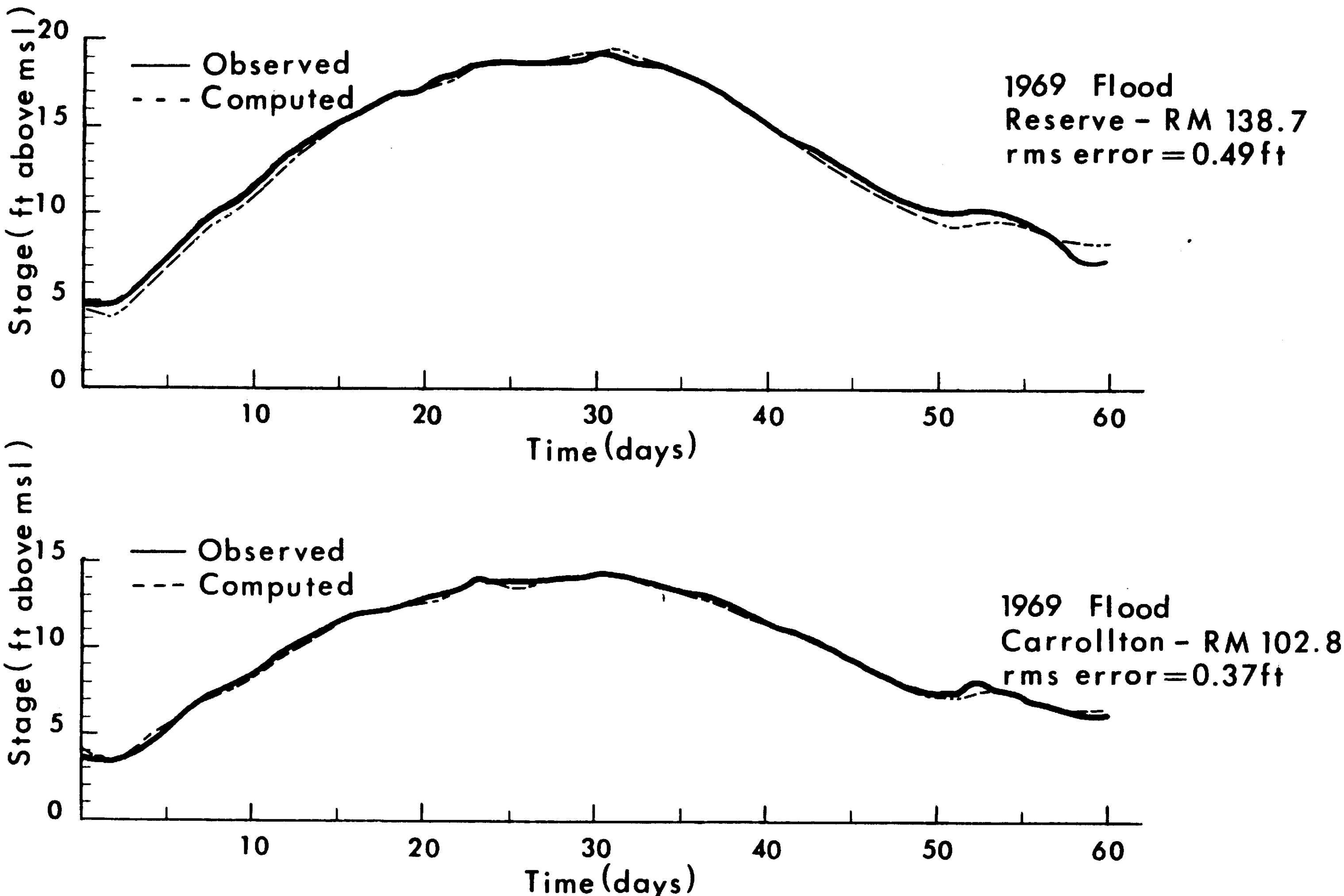
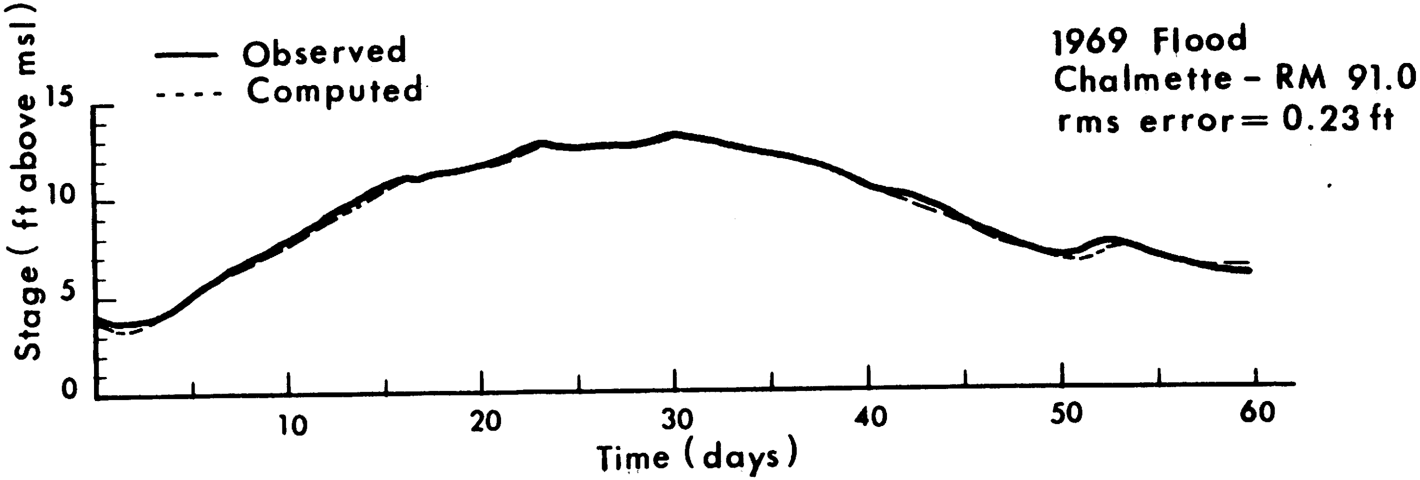


Figure 6. Computed and Observed Stage Hydrographs for 1969 Flood at Reserve and Carrollton.



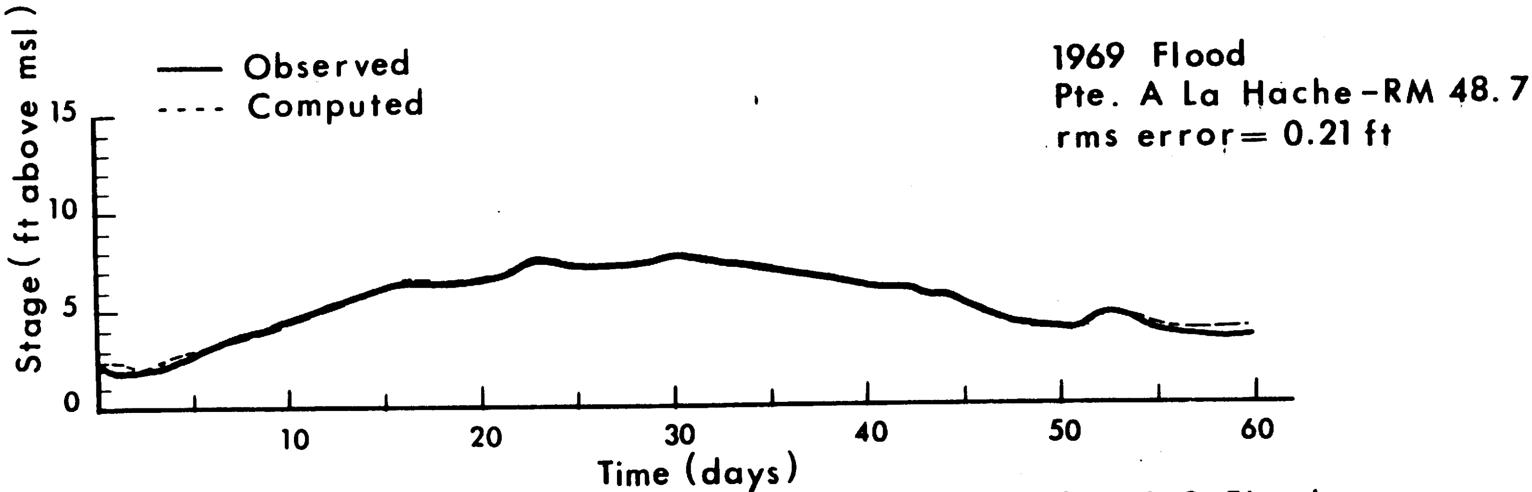


Figure 7. Computed and Observed Stage Hydrographs for 1969 Flood at Chalmette and Pte. A La Hache.

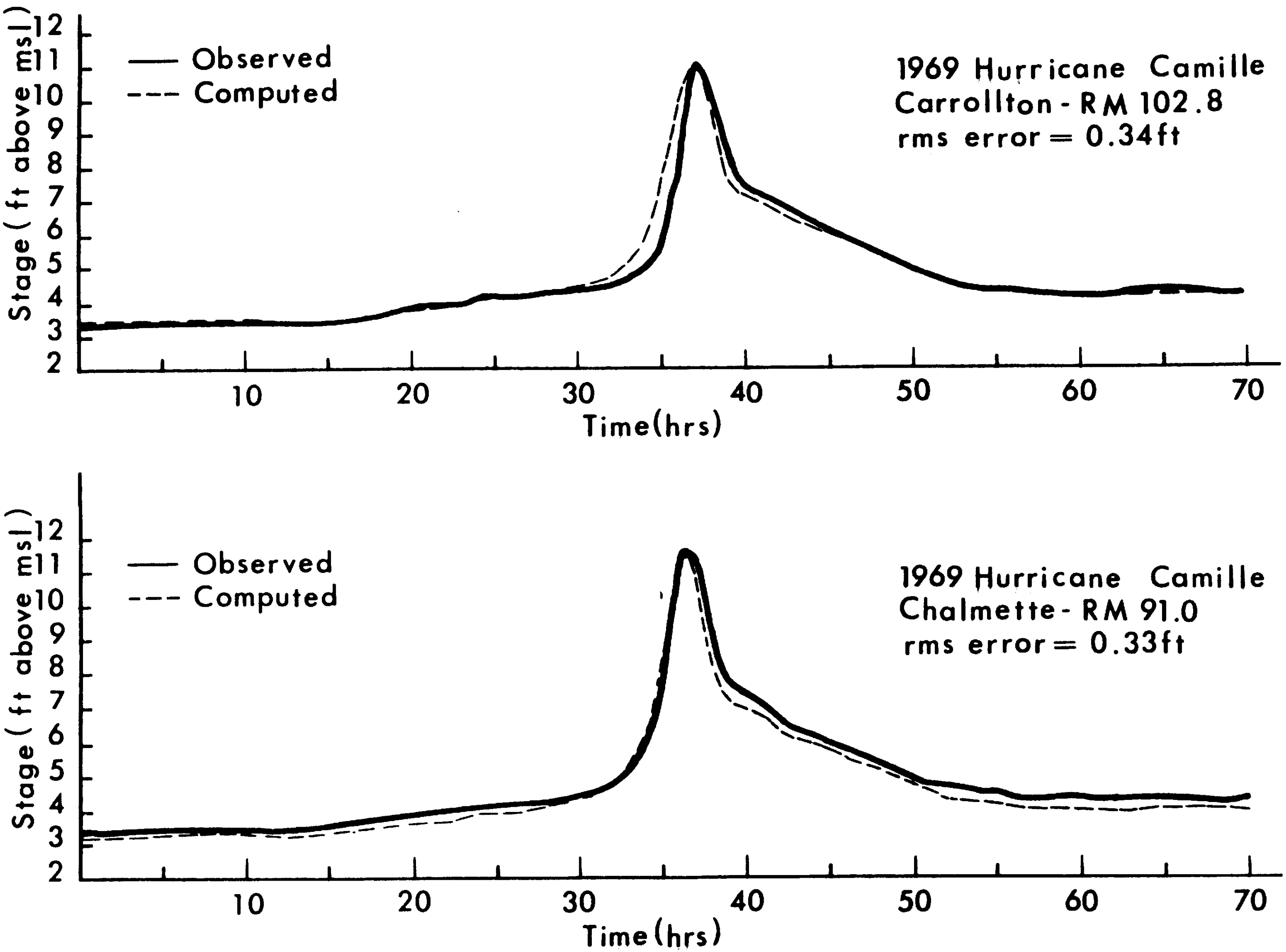


Figure 8. Stage Hydrographs for 1969 Hurricane Camille at Chalmette and Carrollton.

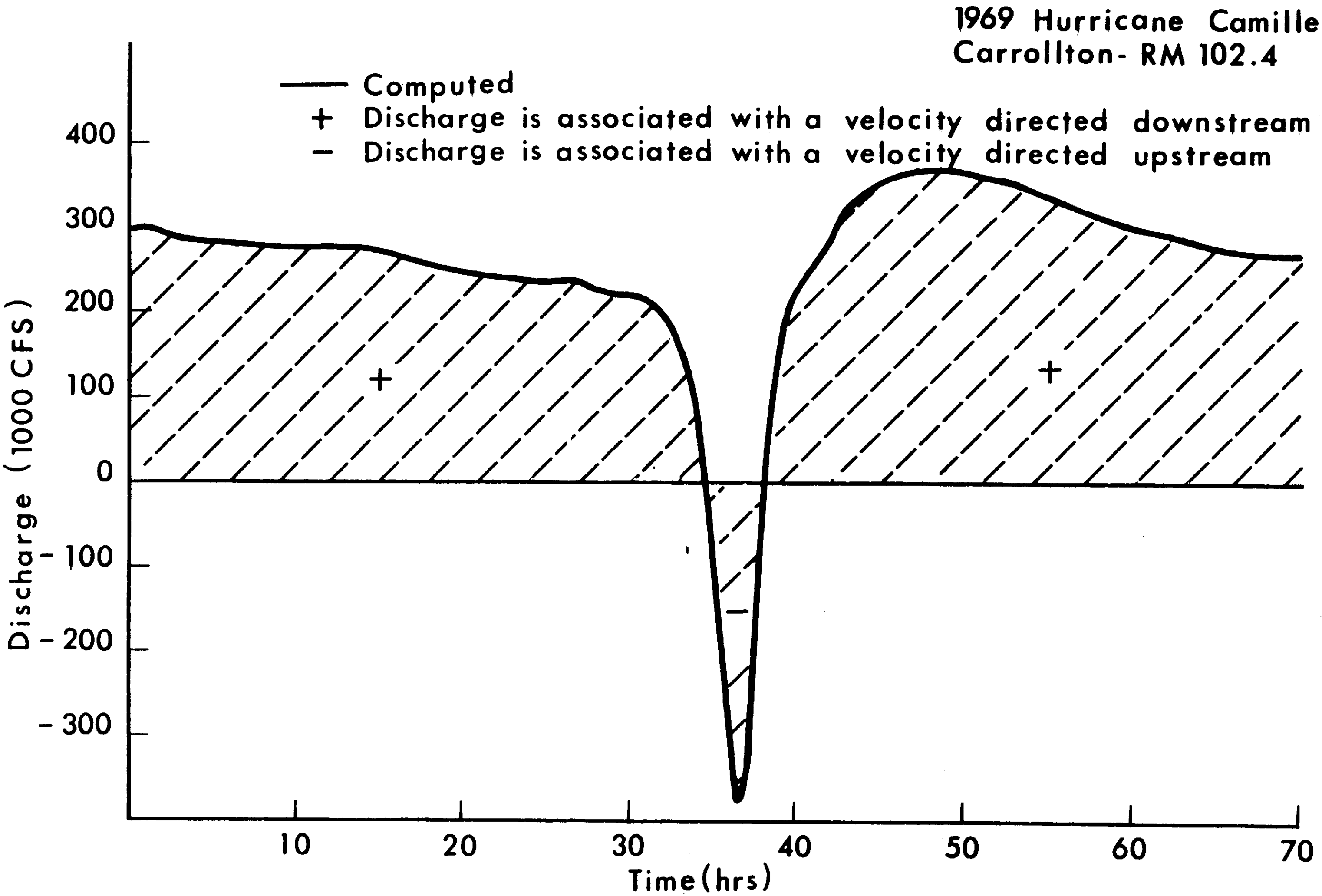


Figure 9. Discharge Hydrograph for 1969 Hurricane Camille at Carrollton.

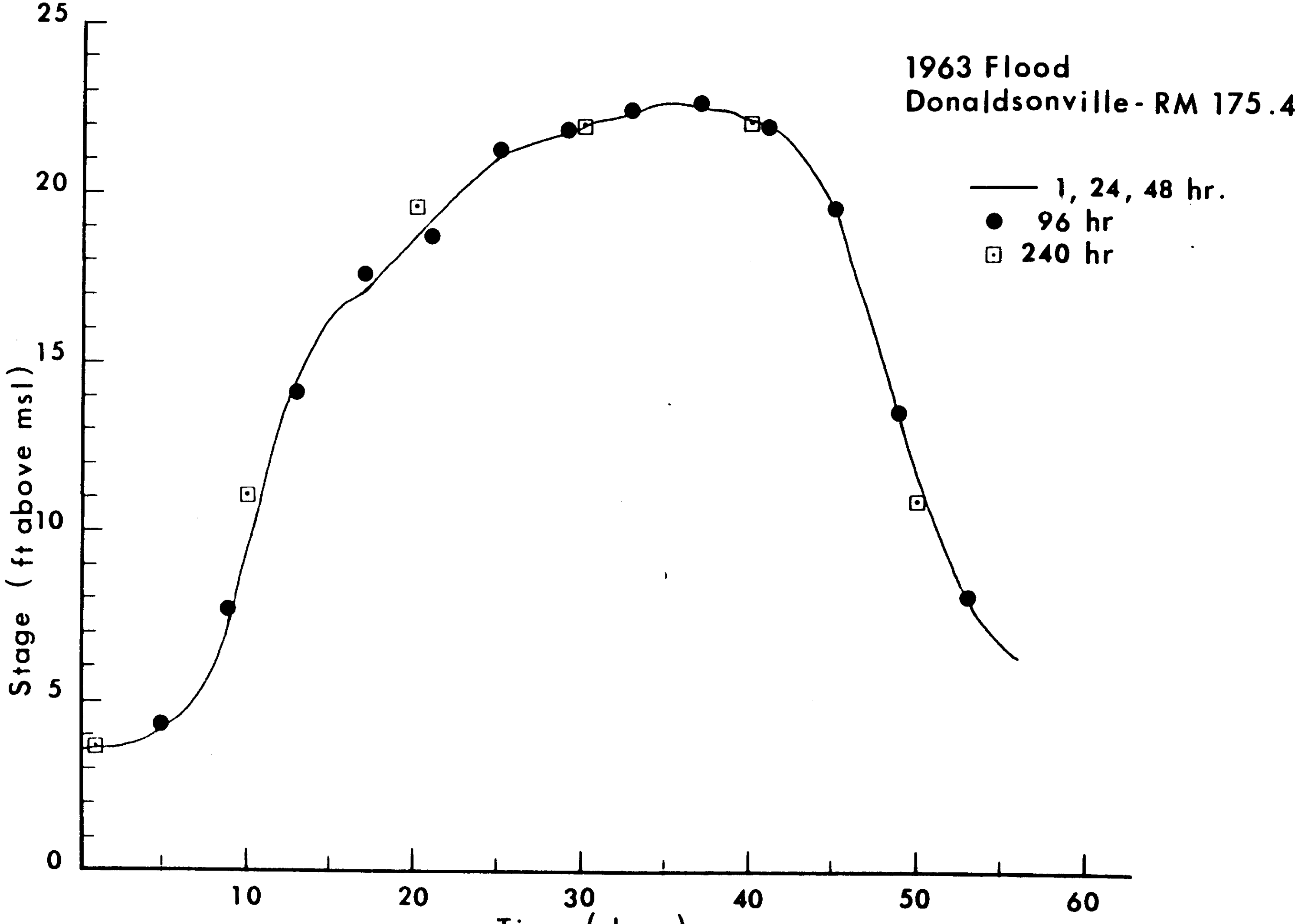


Figure 10. Effect of Δt Time Step Size on Computed Stage Hydrograph at Donaldsonville.

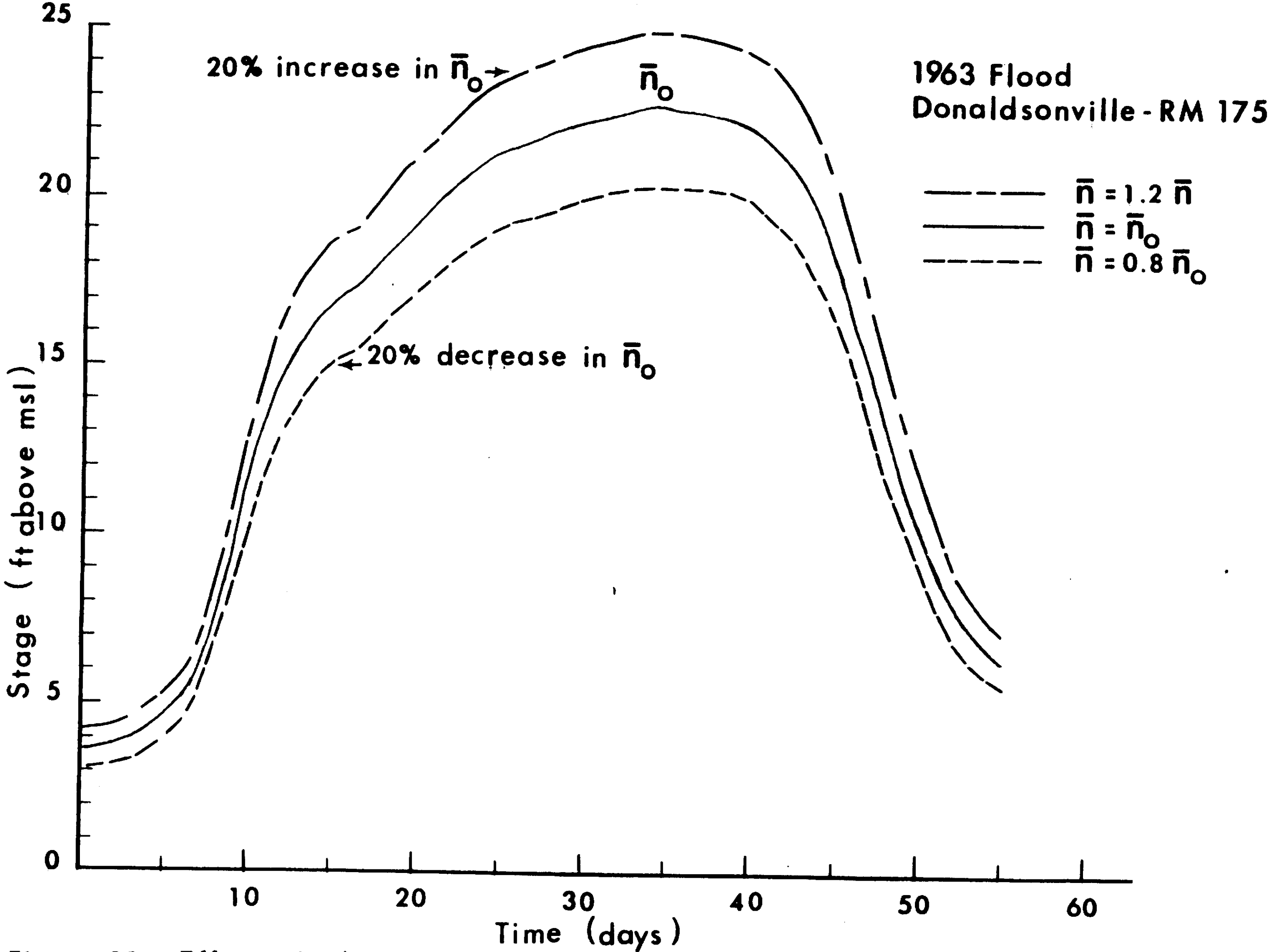


Figure 11. Effect of Changes in Manning's $\bar{n}$ for Stage Hydrograph at Donaldsonville.

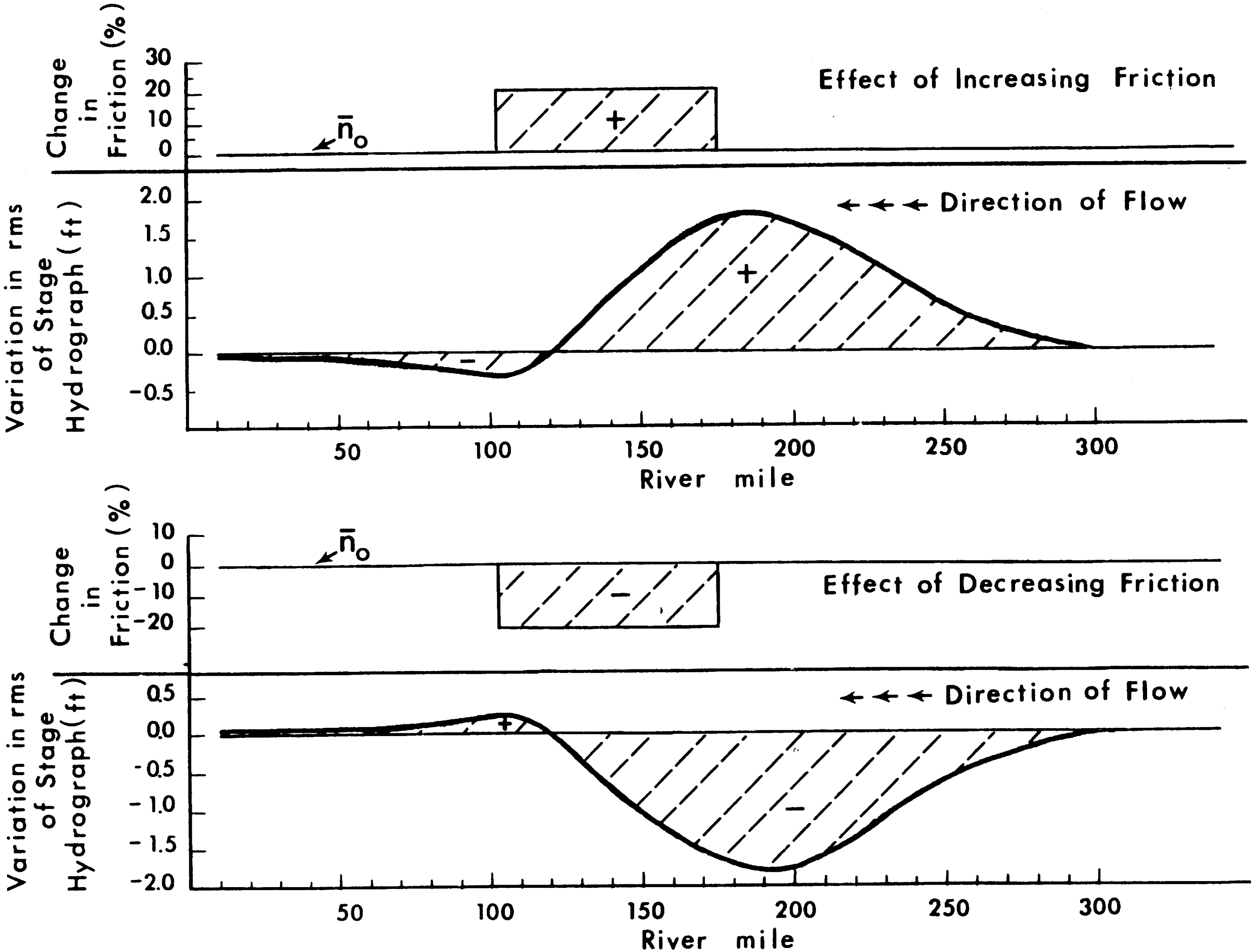


Figure 12. Effects of Increasing or Decreasing Friction on the rms of Computed/Observed Stages.